

Semiannual Progress Report No. 5

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A STUDY OF SELECTED RADIATION AND
PROPAGATION PROBLEMS RELATED TO ANTENNAS
AND PROBE IN MAGNETO-IONIC MEDIA

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INTRODUCTION

This progress report covers the activities in the period starting April 1, 1965 to September 30, 1965. Most research subjects reported earlier are continued. A few preliminary results have been reported and published in various journals (see later). Professor R. Mittra is currently on sabbatical leave, starting September 1, 1965 until February, 1966.

SUMMARY OF THE RESEARCH

1. Impedance of an Electric Current Loop in an Anisotropic Cold Plasma - G. L. Duff

Since the last report, it was decided to proceed with the theoretical calculations of the input impedance of the loop for the case of the imposed steady magnetic field at an arbitrary orientation to the plane of the loop. The far field studies will be done subsequent to completion of these impedance studies.

Theoretical results for the case of the imposed steady magnetic field normal to the plane of the loop are presently being computed numerically on the IBM 7094 computer using Complex Fortran to evaluate the integrals.

In the coming period the theoretical calculations of impedance are expected to be completed and a study of the far fields initiated.

The theoretical results to date have been published in **ELECTRONICS LETTERS** under the title "Input Impedance of a Small Loop of Uniform Electric Current in an Anisotropic Cold Plasma", which appeared in the July, 1965 issue. In addition, the same paper was presented at the 1965 IEEE International Symposium on Antennas and Propagation in Washington, D. C. on September 1, 1965.

2. Experimental Measurement of Loop Impedance - G. L. Duff

Since the last report, the construction of the experimental apparatus for the measurement of the loop impedance has been completed.

The plasma is created by a steady high voltage discharge in helium at pressure of about 1 mm of Hg. Large current coils surround the plasma chamber and these provide the imposed steady magnetic field which causes the plasma to be anisotropic. Electron temperatures as low as 400°K have been achieved. The loop has been inserted into the plasma and impedance measurements are just getting under way. The results of the measurements will be compared with the numerical results of the theory.

3. Current Distribution and Input Impedance of a Cylindrical Antenna in Anisotropic Plasma - S. W. Lee, Y. T. Lo

The problem of the current distribution on an infinitely long, cylindrical antenna in an anisotropic cold plasma has essentially been solved. Numerical computation is underway and expected to be finished in the near future. Some properties of the current distribution are summarized as follows:

- (1) The current solution is in the form of a one-dimensional integration. The integrand is quite complicated. It consists of a double-valued function with four branch points in some cases and/or a two single-valued function with no branch point in some other cases, depending on the parameters of the plasma. A computer program has been rewritten to perform this integration numerically.
- (2) The solution to this problem is quite general. There is no restriction on the radius of the cylinder, nor on the width of the gap of the generator. However, the convergence in the present procedure of numerical integration is slower for a thin antenna than for a fat antenna. A new approach to thin antennas is currently under study.
- (3) The asymptotic behavior of the current at a large distance from the generator can be analytically determined, which has different forms in the different regions of CMA diagrams. As an example, the current on a VLF-antenna at a large distance from the generator consists of two parts. The first is due to a residue contribution and gives rise to a traveling wave. The second is due to the branch cut integration, and dies out as the inverse square of the distance from the generator.

4. Boundary Value Problems in Plasma - S. W. Lee, Y. T. Lo

In recent years, there has been an increasing activity in the problems of plasma in a bounded region. These problems are not only of academic interest but also of practical interest, since only the finite plasma environment is physically realizable in laboratories.

The classical problem of H-plane bifurcation waveguides with anisotropic plasma has been solved by using a cold plasma model, as noted in Progress Report No. 4. This portion of work was presented at the International Symposia of Antennas and Propagation, Washington, D. C., August, 1965, and it is scheduled for publication in the Canadian Journal of Physics, December, 1965. Numerical

evaluation of the reflection coefficients under various plasma parameters have been completed recently. They should be of practical interest.

In order to study the effect of compressibility of the plasma, the H-plane bifurcation problem is reconsidered by employing a warm plasma model. It is found that the modes of the guide can be obtained by perturbation techniques for warm plasma of reasonably high temperature and the reflection coefficient is determined by solving two Wiener-Hopf equations. Numerical computation of this solution is underway. This part of the work is expected to be completed in the near future.

Another boundary value problem studied in this period is the dipole radiation in a composite medium, which consists of the highly conducting earth, the free space, and the ionosphere which is described by a usual warm plasma model. The slow-wave mode for the earth-ionosphere waveguide is found to propagate at all frequencies, in contrast to the case when the ionosphere is described by a cold plasma model. The fast-wave modes have low-frequency cutoff, and their phase velocity increases as the conductivity of the earth increases. The power radiated by each mode in the ionosphere and in the free space region has been numerically evaluated.

5. Theoretical and Experimental Investigation of Circular Waveguides Filled with Warm Plasma - C. Liang, Y. T. Lo

The source-free field behavior in the rectangular, circular, and parallel-plate waveguides completely filled with compressible plasma had been studied. With the boundary condition that the normal component of the electron velocity vanishes at the guide wall, we have found the following conclusions. First, using the method of "separation of variables", no independent modal solution could be obtained for the rectangular guide. Second, the solutions in a guide obtained by using the warm plasma model did not conform with those of the cold plasma model when the zero temperature limit was taken. Therefore, the two different models are not consistent with each other in a bounded region^{1,2}. From the above conclusions, the validity of the boundary condition is naturally questioned. It appears that a thorough understanding of surface physics of conductors in plasma at microscopic level is imperative in order to resolve this dilemma. However, it is felt that such an undertaking, despite its fundamental importance, is beyond the scope of this study since this would require a major effort and long range planning. It is also found that producing a reasonable steady-state plasma within a

metallic guide is extremely difficult. Therefore, the experimental feasibility of a plasma-filled waveguide is somewhat in doubt.

As a result our primary objective in this investigation is set to study both theoretically and experimentally the wave behaviors inside a glass-lined waveguide filled with a compressible plasma due to a simple source and their effect on the source's impedance. This study would provide us with some valuable experiences in antenna and field measurements in a bounded plasma and an opportunity in learning to control the environment of the laboratory plasma such as homogeneity, temperature, and density. Eventually, it is hoped that the effect of electro-acoustic waves on the antenna can be studied.

For both analytic and practical convenience, a circular guide lined with glass is considered. Except at extremely high temperature, the electron plasma cannot penetrate glass. Analytically, therefore, it is more realistic to impose the electron velocity condition at the plasma-dielectric interface.

Currently, we are investigating the source-free field solution under the boundary conditions that (1) the tangential fields are continuous across the plasma-dielectric interface, (2) the tangential electric field vanishes on the guide wall, and (3) there is no electron flux transport across the dielectric boundary. The plasma medium is described by the fluid model with no particle collision and with no externally applied magnetic field. If the inner radius of the dielectric is "a" and its outer radius is "b", then the general field solutions in the regions:

$$\begin{aligned}
 r \leq a : \quad E_{nz} &= \left\{ A_n \left(\frac{\omega_p^2 \gamma}{n_0 e \epsilon_1 \omega^2} \right) J_n(k_p r) + B_n J_n(k_0 r) \right\} e^{in\phi + \gamma z} \\
 H_{nz} &= C_n J_n(k_0 r) e^{in\phi + \gamma z} \\
 a \leq r \leq b : \quad E_{nz} &= \left\{ D_n J_n(k_2 r) + E_n N_n(k_2 r) \right\} e^{in\phi + \gamma z} \\
 H_{nz} &= \left\{ F_n J_n(k_2 r) + G_n N_n(k_2 r) \right\} e^{in\phi + \gamma z}
 \end{aligned}$$

where ω_p is the plasma frequency of the electron

$$\epsilon_1 = 1 - \omega_p^2 / \omega^2$$

$$k_p^2 = \gamma^2 + \omega^2 \epsilon_1 / v_s^2$$

$$k_0^2 = \gamma^2 + \omega^2 \mu_0 \epsilon_0 \epsilon_1$$

$$k_2^2 = \gamma^2 + \omega^2 \mu_2 \epsilon_2$$

v_s is the electron thermal velocity.

After applying the boundary conditions, a general determinantal equation is found to be:

$$\begin{aligned} & \left[\frac{1}{k_2} \left(\frac{\gamma_1}{\gamma_3} \right) J_n(k_2 a) - \frac{1}{k_0} J_n'(k_0 a) \right] \left[\gamma^2 \frac{\omega_p^2}{\omega^2} J_n'(k_0 a) J_n(k_p a) + k_p J_n'(k_p a) \left(\frac{\epsilon_0 \epsilon_1}{\epsilon_2} k_2 \left(\frac{\gamma_0}{\gamma_2} \right) J_n'(k_0 a) - k_0 J_n(k_0 a) \right) \right] \\ &= \frac{\eta^2}{a^2} J_n(k_0 a) \left[\frac{\omega_p^2 \mu_0 \epsilon_0 \epsilon_1}{k_0^3} J_n(k_0 a) J_n(k_p a) + \frac{\gamma^2 k_p}{k_0^3 k_2^3} \cdot \frac{\omega^2 \mu_0 (\epsilon_2 - \epsilon_0 \epsilon_1)}{\epsilon_2} \left(\frac{\gamma_0}{\gamma_2} \right) J_n(k_2 a) J_n'(k_p a) \right. \\ & \quad \left. - \frac{\omega_p^2 \mu_0 \epsilon_2}{k_2^3} \left(\frac{\gamma_0}{\gamma_2} \right) J_n'(k_0 a) J_n(k_p a) \right] \end{aligned}$$

where

$$\begin{aligned} \gamma_0 &= N_n(k_2 b) J_n(k_2 a) - J_n(k_2 b) N_n(k_2 a) \\ \gamma_1 &= N_n'(k_2 b) J_n'(k_2 a) - J_n'(k_2 b) N_n'(k_2 a) \\ \gamma_2 &= N_n(k_2 b) J_n'(k_2 a) - J_n(k_2 b) N_n'(k_2 a) \\ \gamma_3 &= N_n'(k_2 b) J_n(k_2 a) - J_n'(k_2 b) N_n(k_2 a) \end{aligned}$$

If the dielectric is very thin $(b-a) \ll 1$, then

$$\begin{aligned} \gamma_0 &\doteq \frac{2(b-a)}{\pi a} \\ \gamma_1 &\doteq \frac{2(b-a)}{\pi a} \left[1 - \frac{\eta^2}{(k_2 a)^2} \right] \\ \gamma_2 &\doteq \frac{-2}{\pi k_2 a} \\ \gamma_3 &\doteq \frac{2(2a-b)}{\pi k_2 a^2} \end{aligned}$$

At this moment we are studying the propagating characteristics of the symmetric mode ($n=0$) which is ϕ -independent. As one may observe from the determinantal equation, two types of wave will propagate in the guide. A computer program will be written in order to calculate the propagation constant (β) at various operating frequencies (ω). Primary interest will be the cutoff frequency of this mode. When the above information is known, we will have a clearer picture of the propagating properties of the waveguide and, thus, of the design parameters for experimental investigation.

6. Anisotropic Waveguides- I. Akkaya

The characteristic equation of a circular waveguide filled with cold plasma in a uniform magnetic field which is parallel to the guide axis has been studied in great detail. Numerical computation for the propagation constants has been performed for various values of the parameters X , Y , and R which are related to the plasma-frequency, the gyrofrequency and the radius of the guide, respectively. It is found that in the case of gyro-resonance propagation occurs at a radius much larger than that for non-resonance cases.

Normal modes which carry either active power or reactive power for a warm plasma are also considered. In that case the characteristic equation of the guide becomes much more complex. Some solutions are obtained for various values of X , Y , R and W where W is related to the thermal velocity.

The expression for the impedance of an antenna placed inside the guide is formulated for both cold and warm plasma models. For cold plasma cases some results are being evaluated.

Refractive indices of warm anisotropic plasmas using Boltzmann theory are also investigated. An approximate expression for the dispersion relation is obtained for the case

$$\left| \frac{\omega}{c} - \frac{v_t}{\omega_H} n \right| \gg 1$$

for a region of propagation where the angle θ between the propagation vector and the dc magnetic field satisfies the inequality

$$\theta < 65^\circ$$

and also for

$$\theta = 90^\circ,$$

where n is the refractive index. It is found that for

$$\theta < 65^\circ$$

the above inequality for n can be satisfied only if

$$\frac{X}{Y} \gg 1,$$

and in this case the dispersion relation becomes independent of the angle θ .

The solution of the dispersion relation shows that within that angular region

n can never be infinite. For $\theta = 90^\circ$ however, it is found that for values of ω

$$\omega = P \omega_H$$

where P is an integer, in case collisions are neglected, n becomes infinite.

7. Publications Under Grant No. NsG 395 During This Period

1. O. B. Kesler, "Propagation of EM Waves in Linear, Passive, Generalized Media", Antenna Laboratory, Technical Report No. 65-9, University of Illinois, October 1965.
2. G. A. Deschamps, "Angular Dependence of the Refractive Index in the Ionosphere", Radio Science, Vol. 69D, No. 3, March 1965, pp. 395-400.
3. R. Mittra and G. L. Duff, "A Systematic Study of the Radiation Patterns of a Dipole in a Magnetoplasma Based on a Classification of the Associated Dispersion Surfaces", Radio Science, Vol. 69D, No. 5, May 1965, pp. 681-692.
4. G. L. Duff and R. Mittra, "Input Impedance of a Small Loop of Uniform Electric Current in an Anisotropic Cold Plasma", Electronics Letters, July 1965.
5. S. W. Lee and Y. T. Lo, "Radiation Resistance of an Elliptical Loop Antenna with Constant Current in Compressible Plasma", Electronics Letters, July, 1965.
6. S. W. Lee, C. Liang, and Y. T. Lo, "Inconsistency of Boundary Conditions of Plasma Models in a Bounded Region", Electronics Letters, July 1965.
7. S. W. Lee, C. Liang, and Y. T. Lo, "Further Remarks on Boundary Conditions of Plasma Models in a Bounded Region", Electronics Letters, September 1965.
8. S. W. Lee, Y. T. Lo and R. Mittra, "Finite and Infinite Bifurcation Waveguide with Anisotropic Plasma Medium", to appear in the Canadian Journal of Physics, December, 1965.
9. S. W. Lee and Y. T. Lo, "Radiation in an Anisotropic Moving Medium", to appear in Radio Science.

8. Travel

Mr. S. W. Lee attended and presented a paper at the International Symposium of Antennas and Propagation, Washington, D. C., September 1965.

9. Personnel

<u>Name</u>	<u>Position</u>	<u>Percent of full time charge to subject contract</u>
Inci Akkaya	Research Assistant	50
Graham Duff	Research Assistant	50
Oren B. Kesler	Research Assistant (terminated 8/31/65)	50
Y. T. Lo	Associate Professor	20%
R. Mittra	Associate Professor	33 1/3%
C. Liang	Research Assistant	25%
S. W. Lee	Research Assistant	17%